Calc II Final Review #1

1. $\frac{d}{dx}\left[\ln\sqrt{x^2+1}\right] = \frac{\frac{1}{2}(x^2+1)^{-1/2}(2x)}{\sqrt{x^2+1}} = \frac{x}{x^2+1}$

2. $\frac{d}{dx}\left[\log_3(x^2)\right] = \frac{2x}{\ln(3)\,x^2}$

3. $\frac{d}{dx}\left[e^{x^2+2}\right] = 2xe^{x^2+2}$

4. $\frac{d}{dx}\left[3^{\ln(x)}\right] = \frac{\ln(3)3^{\ln(x)}}{x}$

5. $\frac{1}{2}\int \frac{2x}{x^2+3}\,dx = \frac{1}{2}\ln|x^2+3| + C$

6. $\frac{1}{3}\int 3e^{3x}\,dx = \frac{1}{3}e^{3x} + C$

7. $\int \frac{1}{x}3^{\ln x}\,dx = \frac{1}{\ln(3)}3^{\ln(x)} + C$

8. $\frac{1}{4}\int 4\sec(4x)\,dx = \frac{1}{4}\ln|\sec(4x)+\tan(4x)|$

9. $\frac{d}{dx}\left[\arcsin(4x^2)\right] = \frac{8x}{\sqrt{1-16x^4}}$

10. $\frac{d}{dx}\left[\arctan(e^x)\right] = \frac{e^x}{1+e^{2x}}$

11. $\frac{d}{dx}\left[\operatorname{arcsec}(x^2)\right] = \frac{2x}{x^2\sqrt{x^4-1}}$

12. $\int \frac{x+2}{3+4x^2}\,dx = \frac{1}{8}\int \frac{8x}{3+4x^2}\,dx + \int \frac{2}{3+4x^2}\,dx$

$u=2x \quad a=\sqrt{3}$
$du=2dx$

$= \frac{1}{8}\ln|3+4x^2| + \frac{1}{\sqrt{3}}\arctan\left(\frac{2x}{\sqrt{3}}\right) + C$

13. $\frac{d}{dx}\left[\cosh(4x^2)\right] = 8x\sinh(4x^2)$

(14) $\frac{d}{dx}\left[x^2 2^x\right] = x^2 \ln(2) 2^x + 2^x (2x)$

(15) $\frac{1}{2}\int \frac{2x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\arcsin\left(x^2\right) + C$

$u = x^2 \quad a = 1$

$du = 2x\,dx$

(16) $\int \frac{1}{x\ln(x)}\,dx = \ln|\ln(x)| + C$

$u = \ln(x)$

$du = \frac{1}{x}dx$

(17) $\frac{1}{2}\int \frac{2e^{2x}}{1+e^{2x}}\,dx = \frac{1}{2}\ln|1+e^{2x}| + C =$ [redacted]

$u = e^{2x}+1$

$du = 2e^{2x}dx$

(18) $\frac{d}{dx}\left[\operatorname{sech}(x+1)\right] = -\operatorname{sech}(x+1)\tanh(x+1)$

(19) $\frac{d}{dx}\left[x^2 e^{-x}\right] = x^2(-e^{-x}) + e^{-x}(2x)$

$= xe^{-x}(-x+2)$

$= \frac{x(2-x)}{e^x}$

(20) $-\int \frac{-\csc^2 x}{\cot x}\,dx = -\ln|\cot x| + C$

(21) $\frac{d}{dx}\left[\ln(\ln(x))\right] = \frac{1}{x\ln(x)}$

(22) $\int \frac{x-3}{x^2+1}\,dx = \frac{1}{2}\int \frac{2x}{x^2+1}\,dx - 3\int \frac{dx}{x^2+1}$

$u = x \quad a = 1$

$du = dx \qquad = \frac{1}{2}\ln|x^2+1| - 3\arctan(x) + C$

(23) $\int \frac{3^{2x}}{1+3^{2x}}\,dx = \frac{1}{2\ln(3)}\ln|1+3^{2x}| + C$

$u = 3^{2x}$

$du = \ln(3)3^{2x}(2)dx$

(24) $\int \frac{\arcsin x}{\sqrt{1-x^2}}\,dx = (\arcsin x)^2 + C$

$u = \arcsin x$

$du = \frac{1}{\sqrt{1-x^2}}dx$

(25) $\frac{d}{dx}\left[\operatorname{arccot}\sqrt{x}\right] = \frac{-1}{2\sqrt{x}(1+x)}$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}$

ACE II Final Review #2.

① $y = x\sqrt{x^2-3}$

$\ln y = \ln(x) + \frac{1}{2}\ln(x^2-3)$

$\frac{y'}{y} = \frac{1}{x} + \frac{2x}{2(x^2-3)}$

$y' = x\sqrt{x^2-3}\left[\frac{1}{x} + \frac{x}{x^2-3}\right]$

② $\int \frac{x^3-3x^2+5}{x-3}\,dx = \int x^2\,dx + \int \frac{5}{x-3}\,dx$

$= \frac{1}{3}x^3 + 5\ln|x-3| + C$

③ $\int \frac{dx}{x^2+4x+13} = \int \frac{dx}{(x+2)^2+9}$

$= \frac{1}{3}\arctan\left(\frac{x+2}{3}\right) + C$

④ $\int \frac{3}{x^2+x-2}\,dx = \int \frac{1}{x-1}\,dx - \int \frac{1}{x+2}\,dx$

$= \ln|x-1| - \ln|x+2| + C$

⑤ $\int x^4 \ln x\,dx = \frac{1}{5}\ln(x)x^5 - \int \frac{1}{5}x^4\,dx$

$u = \ln(x) \quad dv = x^4\,dx$ $\quad = \frac{1}{5}x^5\ln(x) - \frac{1}{25}x^5 + C$

$du = \frac{1}{x}dx \quad v = \frac{1}{5}x^5$

⑥ $\int \cos^3 x \sin^4 x\,dx = \int (1-\sin^2 x)\sin^4 x\cos x\,dx$

$= \int (\sin^4 x - \sin^6 x)\cos x\,dx$

$= \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C$

⑦ $\int \frac{\sqrt{25-x^2}}{x}\,dx = \int \frac{25\cos^2\theta}{5\sin\theta}\,d\theta$

$= 5\sin\theta$ $\quad = 5\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$

$= 5\cos\theta\,d\theta$

$5 - x^2 = 5\cos\theta$ $\quad = 5\left[\int \csc\theta\,d\theta - \int \sin\theta\,d\theta\right.$

$= -5\ln|\csc\theta + \cot\theta| + 5\cos\theta + c$

$= -5\ln\left|\frac{5}{x} + \frac{\sqrt{25-x^2}}{x}\right| + \sqrt{25-x^2} + c$

⑧ $\int x^2\cos x\,dx = x^2\sin x + 2x\cos x - 2\sin x + C$

+	x^2	$\cos x$
−	$2x$	$\sin x$
+	2	$-\cos x$
−	0	$-\sin x$

⑨ $\int \frac{x+2}{x^2-4x}\,dx = \int \frac{3/2}{(x-4)}\,dx - \int \frac{1/2}{x}\,dx$

$= \frac{3}{2}\ln|x-4| - \frac{1}{2}\ln|x| + C$

⑩ $\int \frac{x+5}{\sqrt{9-(x-3)^2}} = -\frac{1}{2}\int \frac{-2(x-3)}{\sqrt{6x-x^2}}\,dx + \int \frac{8}{\sqrt{9-(x-3)^2}}\,dx$

$u = 6x - x^2$

$du = (6-2x)dx$

$= -2(x-3)dx$

$= -\left(\sqrt{6x-x^2}\right) + 8\arcsin\left(\frac{x-3}{3}\right) + C$

⑪ $\frac{d}{dx}\left[\cosh(x^2+x)\right] = \sinh(x^2+x)(2x+1)$

⑫ $\int \frac{5}{\sqrt{9-x^2}}\,dx = 5\arcsin\left(\frac{x}{3}\right) + C$

⑬ $\int \frac{2x-5}{x^2+2x+2}\,dx = \int \frac{2x-5}{(x+1)^2+1}\,dx$

$= \int \frac{2x+2}{x^2+2x+2}\,dx - \int \frac{5+2}{(x+1)^2+1}\,dx$

$u = x^2 + 2x$

$du = (2x+2)dx$

$= \ln|x^2+2x+2| - 7\arctan(x+1) + C$

(14) $$\int \frac{9x^3}{\sqrt{1+x^2}}\,dx = \int \frac{9\tan^3\theta \sec^2\theta}{\sec\theta}\,d\theta$$

$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\sqrt{1+x^2} = \sec\theta$

$$= \int 9\tan^3\theta \sec\theta\, d\theta$$

$$= \int 9\tan^2\theta(\sec\theta\tan\theta)\,d\theta$$

$$= 9\int \sec^2\theta \sec\theta\tan\theta\, d\theta - 9\int \sec\theta\tan\theta\, d\theta$$

$$= 3\sec^3\theta - 9\sec\theta + C$$

$$= 3(1+x^2)^{3/2} - 9(1+x^2)^{1/2} + C$$

$$= 3(1+x^2)^{1/2}(x^2-2)$$

(15) $$\int \tan^2 x \sec^2 x\, dx = \frac{1}{3}\tan^3 x + C$$

$u = \tan x$

$du = \sec^2 x\, dx$

(16) $$y = \frac{x^2}{(x^3-2)^2}$$

$$\ln y = 2\ln(x) - 2\ln(x^3-2)$$

$$\frac{y'}{y} = \frac{2}{x} - \frac{6x^2}{x^3-2}$$

$$y' = \frac{x^2}{(x^3-2)^2}\left[\frac{2}{x} - \frac{6x^2}{x^3-2}\right]$$

(17) $$\int x\sin^2 x\, dx = \frac{1}{2}\int x(1-\cos(2x))\,dx$$

$$= \frac{1}{2}\int x\, dx - \frac{1}{2}\int x\cos(2x)\,dx$$

$$= \frac{1}{4}x^2 - \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x) + C$$

(18) $$\frac{d}{dx}\left[\operatorname{arcsec}(2x)\right] = \frac{2}{|2x|\sqrt{4x^2-1}}$$

(19) $$\int \cos^2(3x)\,dx = \frac{1}{2}\int [1+\cos(6x)]\,dx$$

$$= \frac{1}{2}x + \frac{1}{12}\sin(6x) + C$$

(20) $$y = x^{x-1}$$

$$\ln y = (x-1)\ln(x)$$

$$\ln y = x\ln(x) - \ln(x)$$

$$\frac{y'}{y} = \frac{x}{x} + \ln(x) - \frac{1}{x}$$

$$y' = x^{x-1}\left[1 + \ln(x) - \frac{1}{x}\right]$$

(21) $$\int \frac{2x^2+7x-3}{x-2}\,dx = \int (2x+11)\,dx + \int \frac{19}{x-2}\,dx$$

$$= x + 11x + 19\ln|x-2| + C$$

(22) $$\frac{d}{dx}\left[\log_5 \sqrt{x^2-1}\right] = \frac{2x}{2\ln(5)(x^2-1)}$$

(23) $$\int \sec^3 x\, dx = \int \sec x(1+\tan^2 x)\,dx$$

$$= \int \sec x\, dx + \int \sec x \tan^2 x\, dx$$

$u = \tan x$ $\quad dv = \sec x\tan x\, dx$

$du = \sec^2 x\, dx$ $\quad v = \sec x$

$$= \int \sec x\, dx + \sec x\tan x - \int \sec^3 x\, dx$$

$$= \frac{1}{2}\left[\ln|\sec x + \tan x| + \sec x\tan x\right]$$

ACE II - Final Review 3.

① $\lim_{x\to 2}\frac{x^2-x-2}{x-2} = \lim_{x\to 2}\frac{2x-1}{1} = \boxed{3}$

② $\int_1^\infty \frac{1}{x^2}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2}\,dx$

$= \lim_{b\to\infty}\left[-\frac{1}{x}\right]_1^b = \lim_{b\to\infty}\left(-\frac{1}{b}+1\right) = \boxed{1}$

③ $\lim_{n\to\infty}\frac{(n-2)!}{n!} = \lim_{n\to\infty}\frac{1}{n(n+1)} = 0$

⇒ converges.

④ $\sum_{n=1}^{\infty}\frac{3n-1}{2n+1}$ $\lim_{n\to\infty}\frac{3n-1}{2n+1} = 3/2 \neq 0$

∴ Series diverges.

⑤ $\sum_{n=1}^{\infty}\frac{4}{2^n} = \sum_{n=1}^{\infty}4\left(\frac{1}{2}\right)^n$ $\left|\frac{1}{2}\right|<1$

Converges.

⑥ $\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+2}\right) = \left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\cdots$
$+\left(\frac{1}{n}+\frac{1}{n+2}\right)+\cdots = \left(1+\frac{1}{2}\right)+\left(\frac{1}{n+1}+\frac{1}{n+2}\right)$
$= \frac{3}{2}+\frac{1}{n+1}+\frac{1}{n+2} = S_n$

$\lim_{n\to\infty} S_n = 3/2$ converges.

⑦ $\sum_{n=1}^{\infty}\frac{4}{\sqrt[4]{n^3}}$ $p = 3/4 < 1$ Diverges.

⑧ $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ ① $\lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$ ✓
② $\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$ ✓
converges.

⑨ $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ $\int_1^\infty \frac{1}{x^2+1}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x^2+1}\,dx$

$f(x) = (x^2+1)^{-1}$ $= \lim_{b\to\infty}\arctan(x)\Big|_1^b = \frac{\pi}{2}-\frac{\pi}{4} = \boxed{\pi/4}$

$f'(x) = -(x^2+1)^{-2}(2x)$

$= \frac{-2x}{(x^2+1)^2} < 0$ for $x \ge 1$. Converges.

⑩ $\sum_{n=1}^{\infty}\frac{n!}{n3^n}$ $\left|\frac{(n+1)!}{(n+1)3^{n+1}}\cdot\frac{n3^n}{n!}\right| = \left|\frac{n(n+1)}{(n+1)3}\right|$

$\lim_{n\to\infty}\left|\frac{n}{3}\right| = \infty$ diverges.

⑪ $\sum_{n=1}^{\infty}\frac{(-1)^n}{(\ln n)^n}$ $\sqrt[n]{\left|\frac{1}{\ln(n)^n}\right|} = \left|\frac{1}{\ln(n)}\right|$

$\lim_{n\to\infty}\left|\frac{1}{\ln(n)}\right| = 0$ Converges

⑫ $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}-1}$ $\frac{1}{\sqrt{n}} \le \frac{1}{\sqrt{n}-1}$ for $n \ge 2$

$\sum\frac{1}{\sqrt{n}}$ diverges thus Series diverges

⑬ $\sum_{n=1}^{\infty}\frac{2}{3^n-5}$ $\sum\left(\frac{1}{3}\right)^n$ converges.

$\lim_{n\to\infty}\left|\frac{2}{3^n-5}\cdot\frac{3^n}{1}\right| = 2$ ← finite, positive
Converges.

⑭ $\sum_{n=1}^{\infty}\frac{(-1)^n n}{n^3-1}$ $\sum_{n=1}^{\infty}\frac{n}{n^3-1}$ compare to $\sum\frac{1}{n^2}$

⇒ Converges absolutely. $\lim_{n\to\infty}\left|\frac{n}{n^3-1}\cdot\frac{n^2}{1}\right| = 1$

⇒ converges

⑮ $\int_1^\infty \frac{\ln(x)}{x}\,dx = \lim_{b\to\infty}\int_1^b \frac{1}{x}\ln(x)\,dx = \lim_{b\to\infty}\frac{1}{2}[\ln(x)]^2\Big|_1^b$

$= \lim_{b\to\infty}\frac{1}{2}(\ln(b))^2 = \infty$ diverges.

⑯ $\lim_{x\to\infty}\frac{5x^2-3x+1}{3x^2-4} = \boxed{\frac{5}{3}}$

⑰ $\sum_{n=1}^{\infty}\frac{(n!)^2}{(3n)!}$ $\left|\frac{[(n+1)!]^2}{(3(n+1))!}\cdot\frac{(3n)!}{(n!)^2}\right|$

$= \left|\frac{(n+1)^2}{(3(n+1))^2}\right| = \left|\frac{(n+1)^2}{9(n+1)^2}\right| = \frac{1}{9}$

$\lim_{n\to\infty}\frac{1}{9} = \frac{1}{9}$ converges.

(18) $\int \frac{\sqrt{1-x^2}}{x^4}\,dx = \int \frac{\cos^2\theta\, d\theta}{\sin^4\theta}$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\sqrt{1-x^2} = \cos\theta$

$= \int \frac{\cos^2\theta}{\sin^2\theta \,\sin^2\theta}\, d\theta$

$= \int \cot^2\theta \csc^2\theta\, d\theta$

$u = \cot\theta \quad du = \csc^2\theta$

$= -\frac{1}{3}\cot^3\theta + C$

$= -\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + C$

(19) $\lim_{x\to\infty} \frac{(\ln x)^2}{x^3} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2\ln x}{3x^3} = \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{2}{9x^3} = 0$

(20) $\sum_{n=1}^{\infty} \frac{3^n}{n^2}$ $\quad \left|\frac{3^{n+1}}{(n+1)^2}\cdot\frac{n^2}{3^n}\right| = \left|\frac{3n^2}{(n+1)^2}\right|$

$\lim_{n\to\infty}\left|\frac{3n^2}{(n+1)^2}\right| = 3 > 1$ Diverges.

(21) $\int_0^1 \frac{1}{(x-1)^{2/3}}\,dx = \lim_{b\to 1^-}\int_0^b \frac{1}{(x-1)^{2/3}}\,dx$

$= \lim_{b\to 1^-} 3(x-1)^{1/3}\Big|_0^b$

$= \lim_{b\to 1^-} 3\left((b-1)^{1/3} - (-1)^{1/3}\right)$

$= 3$ Converges.

(22) $\int \sin^3 x\cos^4 x\,dx = \int (1-\cos^2 x)\cos^4 x \sin x\,dx$

$= \int -\cos^4 x \sin x\,dx + \int -\cos^6 x \sin x\,dx$

$= -\frac{1}{5}\cos^5 x + \frac{1}{7}\cos^7 x + C$

(23) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n+4}}$

$\sum \frac{1}{\sqrt{n+4}}$ compare to $\sum \frac{1}{\sqrt{n}}$ diverges.

$\lim_{n\to\infty} \frac{\sqrt{n}}{\sqrt{n+4}} = 1 \Rightarrow$ diverges.

converges ① $\lim_{n\to\infty} \frac{1}{\sqrt{n+4}} = 0$ ✓

② $\frac{1}{\sqrt{n+4}} \le \frac{1}{\sqrt{n}}$; $n \ge 1$

∴ Converges conditionally.

(24) $\frac{d}{dx}\left[3\arccos\left(\frac{x}{2}\right)\right] = \frac{-3\left(\frac{1}{2}\right)}{\sqrt{1-\left(\frac{x^2}{4}\right)}}$

$= \frac{-3}{\sqrt{4-x^2}}$

(25) $a_n = \frac{3n^2(2n)!}{5(2n+2)!} = \frac{3n^2}{5(2n+2)(2n+1)}$

$\lim_{n\to\infty} a_n = \frac{3}{5}$ converges.

ACE II Final Review # 4

① $f(x) = \ln(x+1)$ $\quad f(0) = 0$

$f'(x) = \frac{1}{x+1}$ $\quad f'(0) = 1$

$f''(x) = \frac{-1}{(x+1)^2}$ $\quad f''(0) = -1$

$f'''(x) = \frac{2}{(x+1)^3}$ $\quad f'''(0) = 2$

$f^{(4)}(x) = \frac{-6}{(x+1)^4}$ $\quad f^{(4)}(0) = -6$

$P_4(x) = 0 + x - \frac{1}{2!_0}x^2 + \frac{2}{3!_0}x^3 - \frac{6}{4!_0}x^4$

② $f(x) = \frac{3}{x+2} = \frac{3}{2-(-x)} = \frac{3/2}{1-(\frac{-x}{2})}$

$f(x) = \sum_{n=1}^{\infty} \frac{3}{2}(-1)^n\left(\frac{x}{2}\right)^n = \sum_{n=1}^{\infty} \frac{3(-1)^n}{2^{n+1}}x^n$

③ $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^n}$ $\quad \left|\frac{x}{2}\right| < 1$

$|x| < 2$

$\boxed{R = 2.}$

④ $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{(n+1)(n+2)}$ $\quad \left|\frac{x^{n+1}}{(n+2)(n+3)} \cdot \frac{(n+1)(n+2)}{x^n}\right| = \left|\frac{x(n+1)}{(n+3)}\right|$

$\lim_{n\to\infty} \left|\frac{x(n+1)}{(n+3)}\right| = |x| < 1$

$x=-1$: $\sum \frac{(-1)^n(-1)^n}{(n+1)(n+2)} = \sum \frac{1}{(n+1)(n+2)}$ Converges $\left(\sum \frac{1}{n^2}\right)$

$x=1$: $\sum \frac{(-1)^n(1)^n}{(n+1)(n+2)} = \sum \frac{(-1)^n}{(n+1)(n+2)}$ Cnvgs. (Alt. series)

$\boxed{-1 \le x \le 1}$

⑤ $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots + \frac{x^n}{n!} + \dots$

$e^{3x+2} = e^2 e^{3x} = e^2\left(1 + (3x) + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \dots + \frac{(3x)^n}{n!} + \dots\right)$

⑥ $\sum_{n=0}^{\infty} \frac{(3x)^n}{(2n)!_0}$ $\quad \left|\frac{(3x)^{n+1}}{(2n+2)!_0} \cdot \frac{(2n)!}{(3x)^n}\right| = \left|\frac{3x}{(2n+2)(2n+1)}\right|$

$\lim_{n\to\infty} \left|\frac{3x}{(2x+2)(2n+1)}\right| = 0 < 1$ Converges $\boxed{R = \infty}$

⑦ $\sum_{n=0}^{\infty} \frac{4^n}{3^n+1}$ $\quad \left|\frac{4^{n+1}}{3^{n+1}+1} \cdot \frac{3^n+1}{4^n}\right| = \left|\frac{4(3^n+1)}{3^{n+1}+1}\right|$

$\lim_{n\to\infty} \left|\frac{4(3^n+1)}{3(3^n+\frac{1}{3})}\right| = \frac{4}{3} > 1$ Diverges

⑧ $f(x) = \frac{4}{3x+2} = \frac{4}{2-(-3x)} = \frac{4}{8-[-3(x-2)]}$

$= \frac{\frac{1}{2}}{1-[-\frac{3}{8}(x-2)]}$ $\quad a = 1/2$, $r = -\frac{3}{8}(x-2)$

$f(x) = \sum_{n=0}^{\infty} \frac{1}{2}\left(-\frac{3}{8}(x-2)\right)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{2}\left(\frac{3}{8}\right)^n(x-2)^2$

⑨ $\sum_{n=1}^{\infty} \frac{5n-3}{n^2-2n+5}$ $\quad \sum \frac{1}{n}$ diverges

$\lim_{n\to\infty} \left|\frac{5n-3}{n^2-2n+5} \cdot \frac{n}{1}\right| = 5 \Rightarrow$ diverges.

⑩ $\frac{d}{dx}\left[\operatorname{arc\,cot}(x^2)\right] = \frac{-2x}{1+x^4}$

⑪ $f(x) = \sqrt{x}$ $\quad f(1) = 1$

$f'(x) = \frac{1}{2}x^{-1/2}$ $\quad f'(1) = 1/2$

$f''(x) = -\frac{1}{4}x^{-3/2}$ $\quad f''(1) = -1/4$

$f'''(x) = \frac{3}{8}x^{-5/2}$ $\quad f'''(1) = 3/8$

$f^{(4)}(x) = \frac{-15}{16}x^{-7/2}$ $\quad f^{(4)}(1) = \frac{-15}{16}$

$P_4(x) = 1 + \frac{1}{2}(x-1) - \frac{1}{4(2)!}(x-1)^2 + \frac{3}{8(3)!}(x-1)^3 - \frac{15}{16(4)!}(x-1)^4$

⑫ $\int \sqrt{4+9x^2}\,dx = \int \frac{4}{3}\sec\theta^3\,d\theta$

$x = \frac{2}{3}\tan\theta$

$dx = \frac{2}{3}\sec^2\theta\,d\theta$

$\sqrt{4+9x^2} = 2\sec\theta$

$= \frac{2}{3}\left[\ln|\sec\theta + \tan\theta| + \sec\theta\tan\theta\right]$

$= \frac{2}{3}\left[\ln\left|\frac{1}{2}\sqrt{4+9x^2} + \frac{3}{2}x\right| + \frac{3}{4}x\sqrt{4+9x^2}\right]$

(13) $\int_0^1 \frac{1}{x^2}\,dx = \lim_{b\to 0^+}\int_b^1 \frac{1}{x^2}\,dx$

$= \lim_{b\to 0^+}\left[-\frac{1}{x}\right]_b^1 = \lim_{b\to 0^+}\left[-1+\frac{1}{b}\right] = \infty$

(14) $\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \cdots$

$\sin(\pi x) = (\pi x) - \frac{(\pi x)^3}{3!} + \frac{(\pi x)^5}{5!} - \frac{(\pi x)^7}{7!} + \cdots$

(15) $\sum_{n=0}^{\infty} (2n)!\left(\frac{x}{2}\right)^n$ $\quad \left|\frac{(2n+2)!\,x^{n+1}}{2^{n+1}} \cdot \frac{2^n}{(2n)!\,x^n}\right|$

$\lim_{n\to\infty} \left|\frac{(2n+2)(2n+1)x}{2}\right| = \infty$ Diverges. $\Rightarrow \boxed{R=0}$

(16) $y = \sqrt{\frac{x^2-1}{x^2+1}}$

$\ln(y) = \frac{1}{2}\ln(x^2-1) - \frac{1}{2}\ln(x^2+1)$

$\frac{y'}{y} = \frac{2x}{2(x^2-1)} - \frac{2x}{2(x^2+1)}$

$y' = \sqrt{\frac{x^2-1}{x^2+1}}\left[\frac{x}{x^2-1} - \frac{x}{x^2+1}\right]$

(17) $\frac{d}{dx}\left[\log_3(x^4+1)\right] = \frac{4x^3}{\ln(3)(x^4+1)}$

(18) $\sum_{n=1}^{\infty} \frac{(-1)^n 5^n}{8^n} \Rightarrow \sum \left(\frac{5}{8}\right)^n$ Cnvgs.

Cnvgs. absolutely.

(19) $y = \lim_{x\to\infty} x^{\frac{1}{x}} \Rightarrow \lim_{x\to\infty} \ln\left(x^{\frac{1}{x}}\right) = \lim_{x\to\infty} \frac{\ln(x)}{x} : \frac{\infty}{\infty}$

$= \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln y = 0$

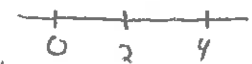
$y = \boxed{1}$

(20) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n2^n}$

$\left|\frac{(x-2)^{n+1}}{(n+1)2^{n+1}} \cdot \frac{n2^n}{(x-2)^n}\right| = \left|\frac{n(x-2)}{2(n+1)}\right|$

$\lim_{n\to\infty}\left|\frac{n(x-2)}{2(n+1)}\right| = \left|\frac{1}{2}(x-2)\right|$

$|x-2| < 2$



$x=0$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-2)^n}{(n+1)(2)^n} = \sum \frac{-1}{n} \Rightarrow$ diverges.

$x=4$: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2^n)}{(n+1)(2^n)} \Rightarrow$ converges.

$\boxed{0 < x \le 4}$

(21) $\int \frac{5^x}{1+5^{x+1}}\,dx = \frac{1}{5\ln(5)}\ln\left|1+5^{x+1}\right| + C$

$u = 1+5^{x+1}$

$du = \ln(5)5^{x+1}dx$

$= 5\ln(5)5^x dx$

ACE II - Final Review #5

① $x = t^2 + 1$

$\boxed{2y^3 + 1 = 3(t^2+1) - 8}$

② $x = 2t^2 \Rightarrow x^2 = 4t^4 \Rightarrow \frac{1}{4}x^2 = t^4$

$y = t^4 + 1$

$y = \left(\frac{1}{4}x^2\right) + 1$

③ $x = t + 1 \qquad x' = 1$

$y = t^2 + 3t \qquad y' = 2t + 3$

$\boxed{\frac{dy}{dx} = \frac{2t+3}{1}}$

$\boxed{\frac{d^2y}{dx^2} = \frac{2}{1}}$

④ $x = t^2$

$y = 2t$

Arc length $= \int_0^2 \sqrt{(2t)^2 + (2)^2}\, dt$

$= \int_0^2 \sqrt{4t^2 + 4}\, dt$

$= \int_0^2 2\sqrt{t^2 + 1}\, dt \approx \boxed{5.92}$

⑤ $3x - y + z = 0$

$3r\cos\theta - r\sin\theta + 2 = 0$

$\boxed{r = \frac{-2}{3\cos\theta - \sin\theta}}$

⑥ $r = 5\cos\theta$

$r^2 = 5r\cos\theta$

$\boxed{x^2 + y^2 = 5x}$

⑦ $r = 2 + \sin\theta$

$\boxed{\begin{aligned} x &= (2+\sin\theta)\cos\theta \\ y &= (2+\sin\theta)\sin\theta \end{aligned}}$

⑧ $r = 3\sin\theta$

$x = 3\sin\theta\cos\theta \qquad x' = -3\sin^2\theta + 3\cos^2\theta$

$y = 3\sin^2\theta \qquad y' = 6\sin\theta\cos\theta$

$\frac{dy}{dx} = \frac{6\sin\theta\cos\theta}{-3\sin^2\theta + 3\cos^2\theta} \Rightarrow \left.\frac{dy}{dx}\right|_{\theta = \pi/3} = \frac{6\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{-3\left(\frac{3}{4}\right) + 3\left(\frac{1}{4}\right)}$

$= \frac{\frac{3\sqrt{3}}{2}}{-\frac{3}{2}} = \boxed{-\sqrt{3}}$

⑨ $r = 5\cos 5\theta$

$0 = 5\cos 5\theta$

$0 = \cos 5\theta$

$5\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{10}$

Area $= 2 \cdot \frac{1}{2}\int_0^{\pi/10} 25\cos^2(5\theta)\, d\theta$

$= \int_0^{\pi/2} 25\cos^2(5\theta)\, d\theta$

$\boxed{= \frac{5\pi}{4}}$

⑩ $\int_1^2 \frac{1}{\sqrt{4 - x^2}}\, dx$

$= \lim_{b \to 2^-} \int_1^b \frac{1}{\sqrt{4-x^2}}\, dx$

$= \lim_{b \to 2^-} \left[\arcsin\left(\frac{x}{2}\right)\right]_1^b$

$= \frac{\pi}{2} - \frac{\pi}{6} = \boxed{\frac{\pi}{3}}$

(11) $x = 1 - t \qquad x' = -1$

$y = t^3 - 3t \qquad y' = 3t^2 - 3$

$$\frac{dy}{dx} = \frac{3t^2-3}{-1} = -3t^2+3$$

$$-3t^2+3=0$$
$$-3t^2=-3$$
$$t^2=1$$
$$\boxed{t=\pm 1}$$

(12) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n}{2n-1}$

$\lim_{n\to\infty} \frac{n}{2n-1} = \frac{1}{2}$ Diverges.

(13) $-\int -e^x \sqrt{1-e^x}\, dx = -\frac{2}{3}(1-e^x)^{3/2} + C$

$u = 1-e^x$

$du = -e^x dx$

(14) $f(x) = \ln(x) \qquad f(2) = \ln(2)$

$f'(x) = \frac{1}{x} \qquad f'(2) = 1/2$

$f''(x) = -\frac{1}{x^2} \qquad f''(2) = \frac{-1}{4}$

$f'''(x) = \frac{2}{x^3} \qquad f'''(2) = \frac{1}{4}$

$$\boxed{P_3(x) = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{4(2!)}(x-2)^2 + \frac{1}{4(3!)}(x-2)^3}$$

(15) $\frac{d}{dx}\left[\tanh\left(\sqrt{x-4}\right)\right] = \frac{1}{2}(x-4)^{-1/2}\operatorname{sech}^2\left(\sqrt{x-4}\right)$

(16) $x = 2+\cos\theta \Rightarrow x-2 = \cos\theta$

$y = 3\sin\theta \Rightarrow \frac{y}{3} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

$$\boxed{(x-2)^2 + \frac{y^2}{9} = 1}$$

(17) $\int x\ln(x+1)\,dx$

$u = \ln(x+1) \qquad dv = x\,dx$

$du = \frac{1}{x+1}dx \qquad v = \frac{1}{2}x^2$

Long ÷

$$= \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx$$

$$= \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\left[\int \frac{1}{x+1}dx + \int x\,dx - \int dx\right]$$

$$\boxed{= \frac{1}{2}x^2\ln(x+1) - \frac{1}{2}\ln(x+1) - \frac{1}{4}x^2 - \frac{1}{2}x + C}$$

(18) $r = 6\sin(2\theta)$

$0 = 6\sin(2\theta)$

$0 = \sin(2\theta)$

$2\theta = 0, \quad 2\theta = \pi$

$\theta = 0, \quad \theta = \pi/2$

$$\text{Area} = \frac{1}{2}\int_0^{\pi/2} 36\sin^2(2\theta)\,d\theta = \boxed{\frac{9\pi}{2}}$$

(19) $r = 3\sin\theta \qquad \frac{1}{6}\sin(2\theta)$

$x = 3\sin\theta\cos\theta$

$$\frac{dx}{d\theta} = -3\sin^2\theta + 3\cos^2\theta$$
$$= 3(\cos^2\theta - \sin^2\theta) = 3\cos(2\theta) = 0$$
$$\cos(2\theta) = 0$$
$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$
$$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}}$$

(20) $f(x) = \frac{1}{2x-5}, \; c=0$

$$= \frac{1}{-5+2(x)} = \frac{1}{-5-(-2x)} = \frac{-\frac{1}{5}}{1-\left(\frac{2}{5}x\right)}$$

$$f(x) = \sum_{n=0}^{\infty}\left(-\frac{1}{5}\right)\left(\frac{2}{5}x\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{-(2^n)}{5^{n+1}}x^n}$$

(21) $\lim_{n\to\infty} x\tan\left(\frac{1}{x}\right) = \lim_{n\to\infty} \frac{\tan\left(\frac{1}{x}\right)}{\frac{1}{x}} = \frac{0}{0}$

$$= \lim_{n\to\infty} \frac{-\frac{1}{x^2}\sec^2\left(\frac{1}{x}\right)}{-\frac{1}{x^2}} = \lim_{n\to\infty} \sec^2\left(\frac{1}{x}\right) = \boxed{1}$$